# Optimization of Oil-Flow Scheduling in Branched Pipeline Systems

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**Abstract:** The article considers a new approach to oil transportation scheduling in oil pipeline network systems. This new approach is based on a mathematical model that solves the optimization transport problem as a system of objective functions and equality and inequality constraints. The system can be varied depending on the needs of a given pipeline system. The approach allows one to compute oil-flow distribution during a certain time period (e.g., day, week, or month) with a given time sampling (e.g., hour, day, or week) considering pipeline characteristics (e.g., flow capacity and technological regimes, among others), oil properties (e.g., mass sulfur fraction and density, among others), and capacity of tank terminals. In addition, the approach enables one to optimize oil transportation in terms of energy consumption. The possibilities of the proposed approach are shown using a real system of 10 oil pipelines, 9 branches, 4 transitional tank terminals, 3 oil suppliers, and 6 oil consumers. The result of the flow distribution calculation in a branched system, which is the schedule of cargo flows for each pipeline in a whole pipeline system with all constraints satisfied and optimized objective function during a 1-month (744 h) time period with a 1-h time sampling, is shown. **DOI:** 10.1061/(ASCE)PS.1949-1204.0000378. 

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## Introduction

The oil pipeline system in many countries (e.g., United States, Russia, and Canada, among others) is a large number of branched pipelines with many consumers and suppliers. The problem of optimizing oil transfer from suppliers to consumers under necessary limitations is a difficult mathematical, engineering, and technical problem for transportation companies.

One of the urgent and time-consuming problems in the area of transportation of oil and oil products is the problem of oil cargo-flow scheduling through trunk pipeline systems. The problem is complicated by the necessity of taking into account both the characteristics of the oil pipelines themselves, the properties of oil, and the capacity of tank terminals. In addition, it is necessary to optimize electricity costs for oil pumping and take scheduled maintenance work into consideration.

Many scientific and engineering works have been devoted to the optimization of energy consumption costs when planning cargo flows for a single main oil pipeline (Sergienko 2012; Economides and Kappos 2009; Zhang and Liang 2016; Wu et al. 2017). In these works, special attention was paid to the selection of pumping equipment at pumping stations, as well as the selection of technological regimes for oil pumping.

For a system consisting of several pipelines, the problem statement is more general, and the choice of technological regimes for pumping in oil pipelines is characterized not only by optimizing the energy costs, but also by the necessity of fulfilling a number of other limitations. For example, it is necessary to coordinate oil cargo flows per time step in tied pipelines and take into account the capacity of the tank terminals, as well as the properties of pumped oil and oil mixing (compounding). More details and problems about oil-flow scheduling in a branched pipeline system were described by Milidiu and dos Santos Liporace (2003) and Grishanin et al. (2016).

Nowadays, the problem of oil-flow scheduling optimization in pipeline systems is very relevant due to

- deterioration of incoming oil quality from suppliers;
- increases in the share of high-sulfur oil from suppliers; and
- the need to supply low-sulfur oil to consumers.

In this regard, it is necessary to schedule pipeline cargo flows to provide required properties of oil supplied to consumers (Grishanin et al. 2016).

There are many articles featuring different approaches to oil transportation scheduling in pipeline systems. In some studies (Arya and Honwad 2015; De la Cruz et al. 2003; Narvaez and Galeano 2004a, b), a genetic algorithm was proposed to solve the problem of oil-flow scheduling optimization, whereas in others (Vlot 2017; Oosterhuis 2015; Wang and Lu 2015; Jamshidifar et al. 2009; Grelli 1985; Osiadacz 1994) dynamic programming was used to optimize pipeline networks. Nowadays, gradient search techniques (Mercado et al. 2002; Rozer 2003; Tabkhi 2007) and heuristic methods (Ferber et al. 1999; Conrado and Rozer 2005) have become quite widespread in optimization of transport of oil and gas pipeline systems. However, the methods used in these techniques have the drawback of getting trapped in local optima. The solution depends on the initial chosen solution, and these methods are not efficient in handling discrete variables.

In addition, there is no consensus among researchers regarding which method of cargo-flow optimization in oil pipeline systems is

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the best. Because this problem is highly relevant in the modern oil and gas field, the authors have developed a new approach to solve the problem. The designed approach allows one to compute the oil-flow schedule for a certain time period (e.g., day, week, or month) with a given time sampling (e.g., hour, day, or week) for all pipelines of a pipeline system.

## Proposed Approach

For detailed oil cargo-flow scheduling through a branched pipeline system, it is necessary to calculate pumping for each pipeline for a given period with a certain sampling of the time steps in such a way that the following conditions are met:

- The capacity and availability of only fixed-regime flows for a pipeline are taken into account (if a pipeline flow is not specified due to planned operations or there is no fixed set of technological regimes for this pipeline).
- The capacity of tank terminals was taken into account (at each time step, the amount of oil in tank terminals did not exceed of the maximum allowable level nor reach the minimum allowable level).
- The total amount of oil taken from the suppliers and transferred to consumers was in accordance with the general transportation schedule.
- Scheduled operations (e.g., planned stops of pumping, extra regimes that are not included in a fixed set of regimes) were taken into account for all pipelines.
- Limitations on the oil properties (e.g., density and mass fraction of sulfur) at specified control points (consumers and tank terminals) were taken into account.
- At junctures of several pipelines (points where flows mix) and in tank terminals, oil was mixed using the weight additivity rule.
- When calculating the oil properties at control points, the transportation of oil was taken into account, that is, the movement of oil with different properties at the flow rate. To calculate oil transportation properties, each pipeline was divided into equal segments, and properties in each segment were computed with accordance with the current flow rate in a pipeline.

The formulas for the oil properties at the juncture of several pipelines have a general form

$$s_n^\theta = \frac{\sum_i s_i^\theta q_{ij}^\theta}{\sum_i q_{ij}^\theta}; \qquad \rho_n^\theta = \frac{\sum_i q_{ij}^\theta}{\sum_i \frac{q_{ij}^\theta}{\rho_i^\theta}} \tag{1}$$

The formulas for oil properties in tank terminals in common form are

$$s_{T_i}^\theta = \frac{s_{T_i}^{\theta-1} M_{T_i}^{\theta-1} + \sum_i s_i^\theta q_{ij}^\theta}{\sum_i q_{ij}^\theta}; \qquad \rho_{T_i}^\theta = \frac{\sum_i q_{ij}^\theta + M_{T_i}^{\theta-1}}{\frac{M_{T_i}^{\theta-1}}{\rho_{T_i}^{\theta-1}} + \sum_i \frac{q_{ij}^\theta}{\rho_i^\theta}} \tag{2}$$

where $\rho_n^\theta$ and $s_n^\theta$ = density and mass fraction of sulfur (%) at the control point at the $\theta$th time step; $M_{T_i}^\theta$ = oil mass in $T_i$th tank terminal at the $\theta$th time step; $q_{ij}^\theta$ = flow rate in the $i$th pipeline for the $j$th technological regime at the $\theta$th time step; and $s_i^\theta$ and $\rho_i^\theta$ = mass fraction of sulfur (%) and density of flowing oil. All notation is listed in the "Notation" section.

Suppose that at first, oil flows into a tank terminal from all supply pipelines. Then oil is mixed, and after that, oil with calculated mixed properties flows from a tank terminal to outlet pipelines.

For the following formulation of the problem, these conditions can be formalized as a system containing an optimized objective function $\varphi(x)$ and constraints $b(x)$, $c(x)$ that are equalities/inequalities.

The objective function has the following general form:

$$\begin{aligned} \varphi(x) &= N_1 f(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, T_{\text{step}}) + N_2 \gamma(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, A_s, B_c, T_{\text{step}}) \\ &\quad + N_3 J(t_{ij}^\theta, T_{\text{step}}), \\ x &= \{t_{ij}^\theta, q_k^\theta, a_s, b_c\} \end{aligned} \tag{3}$$

where $f(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, T_{\text{step}})$ = function of energy costs for oil pumping through the pipeline system; $\gamma(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, a_s, b_c, T_{\text{step}})$ = function of discrepancy between the total amount of oil actually delivered by suppliers and received by consumers from the amount specified by the schedule; $\vartheta(t_{ij}^\theta, T_{\text{step}})$ = function that describes the amount of transitions between regimes in a pipeline system; $T_{\text{step}}$ = duration of the time step; $t_{ij}^\theta$ = pumping time on the $i$th pipeline for the $j$th technological regime at the $\theta$th time step; and $N_1$, $N_2$, and $N_3$ = weighting factors of importance of functions $f(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, T_{\text{step}})$, $\gamma(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, a_s, b_c, T_{\text{step}})$, and $\vartheta(t_{ij}^\theta, T_{\text{step}})$ in the objective function.

Function $\gamma(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, a_s, b_c, T_{\text{step}})$ was added to the objective function to provide a solution of the optimization problem even when it is not possible to transport all planned oil volumes from suppliers to consumers. In that case, the approach tends to minimize the discrepancy between actual and planned transported oil volumes.

The approach suggests arranging weight factors in the following order:

$$0.01 N_2 > N_1 > N_3 \tag{4}$$

In the case in Eq. (4), the first priority is to fulfill the condition that all required amount of oil will be taken from the suppliers and all required amount of oil will be delivered to consumers, if possible. Next, the condition for minimizing energy costs will be fulfilled, and thirdly, the number of transitions between technological regimes is optimized.

According to the specific demands of a certain pipeline system, new terms can be added to the optimized function.

General constraints $b(x)$, $c(x)$ are written as follows:

- $0 \le t_{ij}^\theta \le T_{\text{step}}$ states that the time when a pipeline operates in a certain technological regime must not exceed the duration of a time step $T_{\text{step}}$ and cannot be negative.
- $\sum_j^J t_{ij}^\theta = T_{\text{step}}$ is the sum of operating time for all technological regimes for each step for one pipeline must be equal time step.
- $0 \le q_k^\theta \le Q_k$ indicates that the flow rate in a branch pipeline cannot exceed the branch capacity $Q_k$ and cannot be negative. It is set for branch pipelines that do not have a fixed set of technological regimes.
- $s_c^{\min} \le s_c^\theta \le s_c^{\max}$, $\rho_c^{\min} \le \rho_c^\theta \le \rho_c^{\max}$ states that values of oil properties at consumers must meet constraints.

For constraints at junctures of a pipeline system and in tank terminals, the law of mass balance is used, where

$$\left(\sum_{i=1}^{I}\sum_{j=1}^{J} q_{ij}^\theta t_{ij}^\theta\right)^{in} + \left(\sum_{k=1}^{K} q_k^\theta T_{\text{step}}\right)^{in} = \left(\sum_{l=1}^{L}\sum_{M=1}^{M} q_{lm}^\theta t_{lm}^\theta\right)^{\text{out}} + \left(\sum_{n=1}^{N} q_n^\theta T_{\text{step}}\right)^{\text{out}}$$

states that at junctions, incoming and outcoming oil volumes are equal, and

$$M_{\min}^{T_i} \le \left( M_{T_i}^{\theta-1} + \left( \sum_{i=1}^{I} \sum_{j=1}^{J} q_{ij}^{\theta} t_{ij}^{\theta} \right)^{\text{in}} + \left( \sum_{k=1}^{K} q_k^{\theta} T_{\text{step}} \right)^{\text{in}} \right) - \left( M_{T_i}^{\theta} + \left( \sum_{l=1}^{L} \sum_{M=1}^{M} q_{lm}^{\theta} t_{lm}^{\theta} \right)^{\text{out}} + \left( \sum_{n=1}^{N} q_n^{\theta} T_{\text{step}} \right)^{\text{out}} \right) \le M_{\max}^{T_i}$$

states that oil volume in a tank terminal $M_{T_i}$ must be no less than minimal given value $M_{\min}^{T_i}$ and must be no more than maximal given value $M_{\max}^{T_i}$ at each time step for each tank terminal, and $t_{ij}^{\theta} = T_{\text{plan}}$ indicates that in case of scheduled maintenance work, the pumping time of a regime required for work is fixed.

In addition to the preceding limitations, additional equations can be added to the system in formulating special problems for a particular pipeline system.

The aforementioned objective function and constraints described are general and serve as the basis for the described approach for scheduling oil flows in branched pipeline systems using algorithms to solve the optimization transport problem. General information on the formulation and solution of transport problems has been given by Rachev and Ruschendorf (1998) and Foster (1975). The values of the coefficients in the objective function, as well as the complete set of constraints, will depend on the specific tasks assigned when scheduling oil cargo flows.

## Methods of Transport Problem Optimization

The transport problem can be optimized both by the methods of sequential quadratic programming (Mitradjieva-Daneva 2007; Nocedal and Wright 2006; Gomes 2007) and by linear programming methods (Luenberger and Yinyu 2008; Reeb and Leavengood 2002; Stoer and Bulirsch 1993) depending on the complexity of a branched transport system and given limitations.

For the aforementioned $\varphi(x)$ and $b(x)$, $c(x)$, the optimization problem is solved using linear programming algorithms [for example, the simplex method (Murty 2000) or potential method (Bienstock 2001)]. In the approach proposed here, the GNU Linear Programming Kit (GLPK) software package (Oki 2012) is used to implement simplex method and solve the optimization problem for transport problems in a pipeline system. GNU is a recursive acronym for GNU's not Unix! and is a free operating system and extensive collection of computer software. GLPK uses the revised simplex method (Morgan 1997) and primal-dual interior point method (Bonnans et al. 2006) for noninteger problems and the branch-and-bound algorithm (Land and Doig 1960) together with Gomory's mixed-integer cuts (Cornuéjols 2008) for integer problems. The calculation step is equal to the sampling step (for example, 1 h). The calculation period used was 1 month (744 h).

The results of the calculations are the determination of the technological pumping regimes for all pipelines with a fixed set of regimes at each calculation step as well as the mass pumping flow rates at each step for pipelines that do not have a fixed set of technological regimes.

If it is not possible to find a solution to the problem in the preceding formulation, it is concluded that the pipeline system cannot pump required volumes of oil under given constraints on the oil properties at control points. In this case, one must remove the restrictions on $s_c^{\theta}, \rho_c^{\theta}$ from $b(x)$, $c(x)$ and add two additional functions, $\alpha$ and $\beta$, to the objective function $\varphi(x)$

$$\alpha(S_c^{\theta}, S_c^{\max}, S_c^{\min}) = \sum_{\theta=1}^{T} \sum_{c=1}^{C} e^{N_4(s_c^{\theta} - S_c^{\max})} + e^{N_4(S_c^{\min} - s_c^{\theta})} \quad (5)$$

$$\beta(\rho_c^{\theta}, \rho_c^{\max}, \rho_c^{\min}) = \sum_{\theta=1}^{T} \sum_{c=1}^{C} e^{N_5(\rho_c^{\theta} - \rho_c^{\max})} + e^{N_5(\rho_c^{\min} - \rho_c^{\theta})} \quad (6)$$

Hence, the objective function takes the following form:

$$\begin{aligned} \varphi(x) &= N_1 f(q_{ij}^{\theta}, t_{ij}^{\theta}, q_k^{\theta}, T_{\text{step}}) + N_2 \gamma(q_{ij}^{\theta}, t_{ij}^{\theta}, q_k^{\theta}, T_{\text{step}}) \\ &\quad + N_3 \vartheta(t_{ij}^{\theta}, T_{\text{step}}) + \alpha(N_4, S_c^{\theta}(t_{ij}^{\theta}, q_k^{\theta}), S_c^{\max}, S_c^{\min}) \\ &\quad + \beta(N_5, \rho_c^{\theta}(t_{ij}^{\theta}, q_k^{\theta}), \rho_c^{\max}, \rho_c^{\min}), \\ x &= \{t_{ij}^{\theta}, q_k^{\theta}\} \end{aligned} \quad (7)$$

where $N_4, N_5 \approx$ 10–100. In this form, even a slight deviation of oil properties from restrictions will lead to a significant increase in the value of the objective function.

The linearity/nonlinearity of the objective function and constraints depends on the complexity of the pipeline system under consideration. In the case of a branched pipeline system with a large number of tank terminals and flow mixing points, the objective function Eq. (7) will be nonlinear due to the functions $\alpha$ and $\beta$.

In the proposed model, the sequential quadratic programming (SQP) method (Nocedal and Wright 2006) is used to solve the problem of nonlinear optimization. This method is implemented in the GNU software package as a successive quadratic programming solver («sqp» function) (Eaton et al. 2016). Because the use of quadratic programming methods significantly increases the calculation time of the optimization problem, and the calculation time has an exponential dependence on the number of variables $x$, with the number of variables exceeding ≈105 and the necessity of using the functions $\alpha$ and $\beta$ in the objective function, it is proposed to solve the problem in two stages:

1. In the first stage, the objective function $\varphi(x)$ is similar to Eq. (7). The optimization problem is solved by quadratic programming methods (Nocedal and Wright 2006; Eaton et al. 2016) because the functions $\alpha(S_c^{\theta}, S_c^{\max}, S_c^{\min})$ and $\beta(\rho_c^{\theta}, \rho_c^{\max}, \rho_c^{\min})$ are nonlinear. The time step is equal to the calculation period [for example, 1 month (744 h)]. The result of the first stage is the calculated values of the properties at the control points, which become new constraints for the second stage.
2. In the second stage of the calculation, the objective function $\varphi(x)$ is similar to Eq. (3). The optimization problem is solved using linear programming algorithms via the GLPK software package (for example, the simplex method or method of potentials).

In the second stage, the calculation step is equal to the sampling step (for example, 1 h). The calculation period is, for example, 1 month (744 h). The result of the calculation in the second stage is the determination of the technological regimes on all pipelines that have a fixed set of regimes, as well as the mass-flow rates for pipelines that do not have a fixed set of technological regimes at each time step.

## Assumptions

When creating the approach, the authors made the following assumptions:

- Transient processes (including those caused by cavitation) within the technological section of a pipeline are not taken into account. That is, transition from one steady regime to another is momentary.

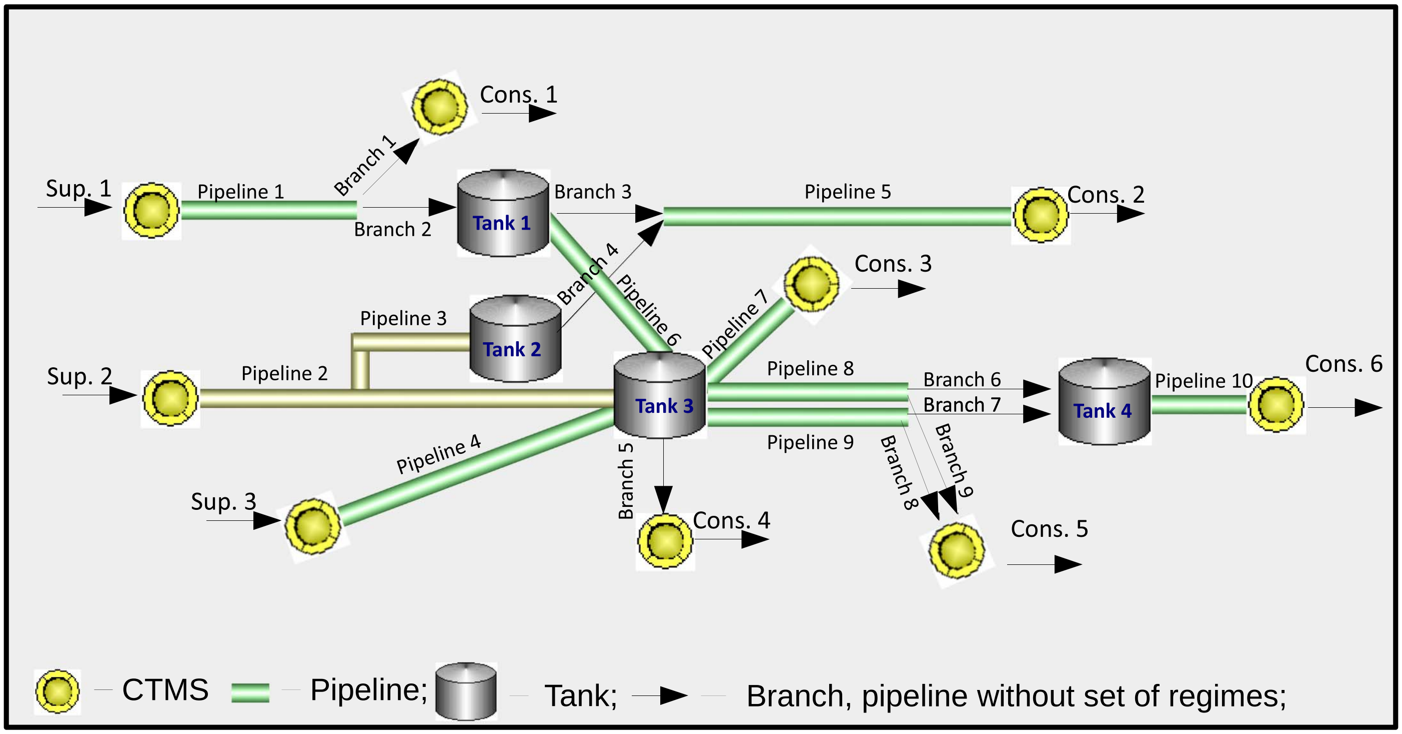


**Fig. 1.** Scheme of the branched oil pipeline system. Cons. = consumer; and Sup. = supplier.

- Transient processes and the changes of the regime mass flow associated with discrete mixing and changes of oil properties, as well as the presence of mixing zones along the entire pipeline, are not taken into account. It is assumed that the regime mass-flow rate is constant and does not depend on the oil properties in a pipe at any time. The assumption that the regime mass-flow rate in a pipe is constant leads to an error in calculating the cargo flow in the pipe associated with the change in density in the pipeline. The magnitude of error depends on the range of density variation in the pipeline system.
- The movement of oil is considered one-dimensional and piston-like, that is, oil with different properties in a pipe is not mixed while pumping through a pipeline.
- Oil temperature and ambient temperature are not taken into account.
- Mixing of oil at the mixing point of flows is considered instantaneous and complete.
- Passage of pigs is not taken into account.
- A tank terminal is not divided into separate tanks and is considered as a single container with oil in which complete uniform mixing of oil occurs when oil enters. One tank terminal can be divided into several groups of tanks. A group can consist of either one or several tanks.

## Results

The results of solving the optimization problem are values of all the variables included in the objective function, i.e., pumping time in various technological regimes ($t_{ij}^{\theta}$) for pipelines, which have a fixed set of technological regimes; mass-flow rate ($q_{ij}^{\theta}$) at each step for pipelines, which does not have a fixed set of technological regimes, as well as volumes of oil that were not transported from suppliers and not delivered to consumers. The determination of these variables will allow calculating the volume of oil in all tank terminals, oil properties at control points, and operating regimes of all pipelines at each time step.

The use of this approach makes it possible to solve the problem with given input data, and if there is no solution with the initial data, it shows how much it is necessary to reduce the traffic flow or change the limitations on the oil properties so that the solution becomes possible. At the same time, the solution is optimized for energy costs.

This suggested approach therefore allows the optimization problem of oil and oil product transportation in a large branched pipeline system to be solved and helps operators to plan technological pumping regimes for a long period.

## Example of Computation of Oil-Flow Schedule in a Real Branched Pipeline System

To test the proposed approach, a real branched oil pipeline system consisting of 10 separate pipelines, 9 branches, 4 tank terminals, 3 suppliers, and 6 consumers was chosen wherein technological regimes of Pipeline 2 and Pipeline 3 are tied. The scheme of the oil pipeline system is shown in Fig. 1.

Pipeline 5 can be supplied from Tank terminal 1 and Tank terminal 2; oil completely mixes at the inlet to the Pipeline 5. Oil properties after mixing are calculated by the mass additivity

**Table 1.** Scheduled gross oil supplies from suppliers and oil properties

| Parameter | Supplier number in Fig. 1 | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Scheduled mass of oil supply, $A_{Si}$ (thousands of t) | 4,150 | 3,200 | 1,430 |
| Mass fraction of sulfur, $S_{Ci}$ (%) | 1.3 | 2.1 | 1.7 |
| Density, $\rho_{Ci}$ (kg/m$^3$) | 845 | 890 | 850 |



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**Table 2.** Scheduled gross oil supplies to consumers and limitations on oil properties

| Parameter | Consumer number in Fig. 1 | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| Scheduled mass of oil consumption, $B_{Ci}$ (thousands of t) | 120 | 4,380 | 1,150 | 1,240 | 400 | 1,500 |
| Limitation on maximum value of mass fraction of sulfur, $S^{Ci}_{\max}$ (%) | 1.9 | 1.9 | 1.9 | 1.9 | 1.9 | 1.9 |
| Limitation on maximum value of density, $\rho^{Ci}_{\max}$ (kg/m$^3$) | 880 | 880 | 880 | 880 | 880 | 880 |

**Table 3.** Initial mass of oil and oil properties in tank terminals and maximum/minimum allowable mass of oil in tank terminals

| Parameter | Tank number in Fig. 1 | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Initial mass of oil, $M^0_{Ti}$ (thousands of t) | 77.69 | 65.45 | 98.175 | 5.95 |
| Mass fraction of sulfur, $S^0_{Ti}$ (%) | 1.22 | 2.15 | 1.67 | 1.7 |
| Density, $\rho^0_{Ti}$ (kg/m$^3$) | 850 | 850 | 850 | 850 |
| Maximum allowable mass, $M^{Ti}_{\max}$ (thousands of t) | 135.75 | 127.5 | 110.5 | 11.48 |
| Minimum allowable mass, $M^{Ti}_{\min}$ (thousands of t) | 0.4 | 0.4 | 0.4 | 0.4 |

**Table 4.** Capacity of branches in pipeline system

| Parameter | Branch number ($k$) in Fig. 1 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Branch capacity, $Q_k$ (thousands of t/h) | 0.42 | 8.33 | 8.43 | 8.43 | 2.29 | 1.04 | 2.5 | 2.5 | 1.04 |

rule in Eq. (1). At the same time, pipes running from Tank terminal 1 and Tank terminal 2 to pipeline 5 do not have fixed technological regimes, so the flow rate during pumping can take any value from zero to the capacity of the pipe. The same holds for pipes from Pipeline 1 to Consumer 1, from Tank terminal 3 to Consumer 4, from Pipeline 8 and Pipeline 9 to Consumer 5, and in Tank terminal 4.

When calculating the test case, a transfer period of 1 month [31 days (744 h)] was chosen with a step increment of 1 h. In the general transportation schedule, the gross pumping volumes for the month received from suppliers and supplied to consumers are set. Limitations on the mass fraction of sulfur and oil density among consumers are as follows. The mass fraction of sulfur for all consumers should not exceed $S^{Ci}_{\max} = 1.9\%$, and density $\rho^{Ci}_{\max} = 880\ \mathrm{kg/m^3}$. Gross volumes of pumping (transportation schedule) and the value of the oil properties from suppliers, as well as the restrictions on the oil properties, are presented in Tables 1 and 2. Properties of oil from suppliers are constant throughout all periods of the calculation.

As initial data, one can specify both the total gross volume of supply and consumption and the distribution of the supply by days or steps, for example, uniform distribution or accurate values (if an accurate schedule of supply and consumption is known), i.e., on what day, how much oil should be taken from suppliers and delivered to consumers. Also, the initial data set consists of oil properties in pipes, amount of oil in tank terminals, properties of this oil at the beginning of the calculation period, and maximum/minimum capacity of tank terminals (Table 3). Moreover, the maximum and minimum capacity of a tank terminal can vary during the calculation period, for example, due to the withdrawal of some tanks for repairs.

In addition, the capacity was set for the pipelines without fixed technological regimes (Table 4) and technological regimes for all other pipelines (Tables 5 and 6), as well as the schedule of maintenance work. The technological regimes and the capacity of the pipelines are indicated in the graphs of the solution (Figs. 2–4).

The schedule of maintenance work is given in Table 7. The assumption that the mass-flow rate is constant at the indicated range of density changes (Table 1) in the pipeline system would lead to a certain error. For the example given, if the mass-flow rate was estimated for a density of 890 kg/m$^3$ and the actual density of oil was 845 kg/m$^3$, the maximum error in the calculation of cargo flows is about 7%–8%.

If there is maintenance work in a pipeline, then for all period of the maintenance work, $t^{\theta}_{ij} = T_{\text{plan}}$. That is, pumping is in a planned technological regime.

The objective function Eq. (7) in this case has the following form:

$$\varphi(x) = N_1 P^{\theta}_{ij} t^{\theta}_{ij} + N_2 \sum_{s=1}^{S} a_s + N_2 \sum_{c=1}^{C} b_c + t^{\theta}_{ij}(T_{\text{step}} - t^{\theta}_{ij}) N_3 + \sum_{\theta=1}^{T} \sum_{c=1}^{C} (e^{N_4(s^{\theta}_c - S^{\max}_c)} + e^{N_4(S^{\min}_c - s^{\theta}_c)}) + \sum_{\theta=1}^{T} \sum_{c=1}^{C} (e^{N_5(\rho^{\theta}_c - \rho^{\max}_c)} + e^{N_5(\rho^{\min}_c - \rho^{\theta}_c)}),$$
$$x = \{t^{\theta}_{ij}, q^{\theta}_k\} \qquad (8)$$

where $P^{\theta}_{ij} = q^{\theta 3}_{ij}$ is the power capacity of all pumps in pipeline to pump oil in the $i$th pipeline for the $j$th technological regime at the

**Table 5.** Initial properties in pipelines in pipeline system

| Parameter | Pipeline number ($i$) in Fig. 1 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Diameter (m) | 1.02 | 1.02 | 0.72 | 0.8 | 1.02 | 0.82 | 0.82 | 0.53 | 0.72 | 0.53 |
| Length (km) | 442 | 150 | 13.3 | 152 | 344 | 18.4 | 320 | 394 | 394 | 197 |
| Initial mass fraction of sulfur in pipeline (%) | 1.22 | 2.15 | 2.15 | 1.75 | 1.55 | 1.22 | 1.7 | 1.7 | 1.7 | 1.7 |
| Initial density in pipeline (kg/m$^3$) | 850 | 850 | 850 | 850 | 850 | 850 | 850 | 850 | 850 | 850 |

**Table 6.** Flow rate on technological regimes ($j$) in pipelines ($i$) in pipeline system

| Regime number ($j$) in Fig. 1 | Flow rate in Pipe 1 (thousands of t/h) | Flow rate in Pipe 2 (thousands of t/h) | Flow rate in Pipe 3 (thousands of t/h) | Flow rate in Pipe 4 (thousands of t/h) | Flow rate in Pipe 5 (thousands of t/h) | Flow rate in Pipe 6 (thousands of t/h) | Flow rate in Pipe 7 (thousands of t/h) | Flow rate in Pipe 8 (thousands of t/h) | Flow rate in Pipe 9 (thousands of t/h) | Flow rate in Pipe 10 (thousands of t/h) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 1.28 | 1.65 | 0.00 | 1.04 | 2.97 | 0.67 | 0.24 | 0.29 | 0.70 | 0.38 |
| 3 | 2.33 | 1.65 | 1.65 | 1.39 | 3.20 | 0.75 | 1.16 | 0.33 | 1.00 | 0.49 |
| 4 | 2.93 | 2.36 | 2.36 | 1.64 | 3.81 | 0.92 | 1.66 | 0.45 | 1.25 | 0.68 |
| 5 | 3.54 | 2.37 | 0.00 | 1.83 | 4.24 | 1.00 | 2.14 | 0.59 | 1.62 | 0.72 |
| 6 | 4.06 | 2.84 | 2.84 | 1.99 | 4.74 | 1.17 | 8.42 | 0.75 | 1.94 | 0.70 |
| 7 | 4.42 | 2.86 | 0.00 | 2.67 | 5.03 | 1.25 | — | 0.83 | 2.02 | — |
| 8 | 4.72 | 3.28 | 2.50 | 2.71 | 5.35 | 1.42 | — | 0.97 | 2.50 | — |
| 9 | 5.03 | 3.62 | 2.25 | 1.88 | 5.84 | 1.50 | — | 1.04 | — | — |
| 10 | 5.36 | 4.21 | 0.00 | — | 6.11 | 1.67 | — | — | — | — |
| 11 | 5.87 | 4.92 | 2.74 | — | 6.31 | 1.75 | — | — | — | — |
| 12 | 6.26 | 5.75 | 2.71 | — | 6.49 | 1.92 | — | — | — | — |
| 13 | 7.33 | 5.83 | 3.33 | — | 6.99 | 2.00 | — | — | — | — |
| 14 | 8.33 | 5.83 | 0.00 | — | 7.23 | 2.17 | — | — | — | — |
| 15 | — | 2.50 | 0.00 | — | 8.07 | 2.25 | — | — | — | — |
| 16 | — | — | — | — | 8.27 | 2.42 | — | — | — | — |
| 17 | — | — | — | — | 8.38 | 2.50 | — | — | — | — |
| 18 | — | — | — | — | 8.43 | 6.25 | — | — | — | — |

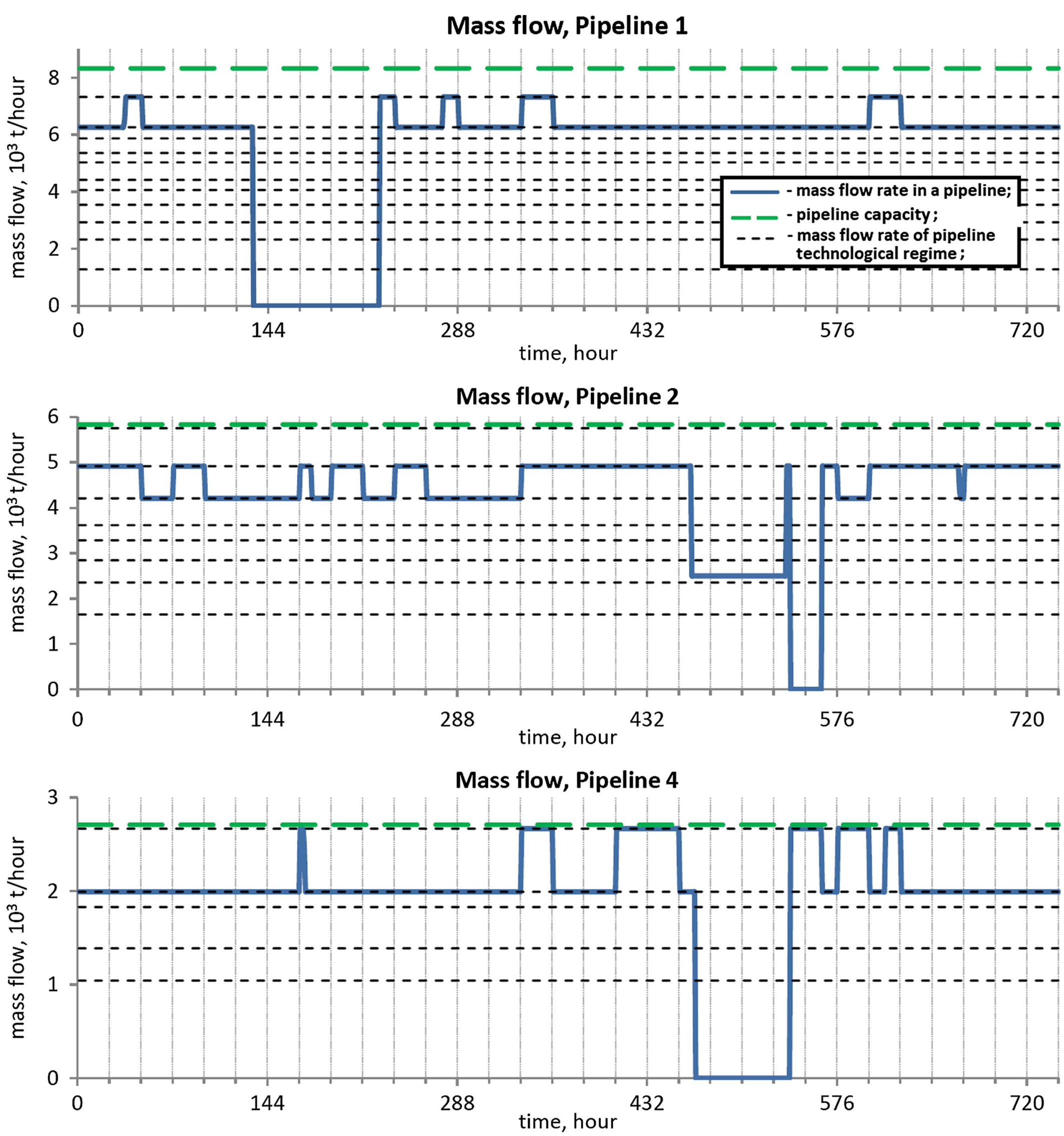


**Fig. 2.** Oil mass-flow rates in pipelines coming from suppliers.

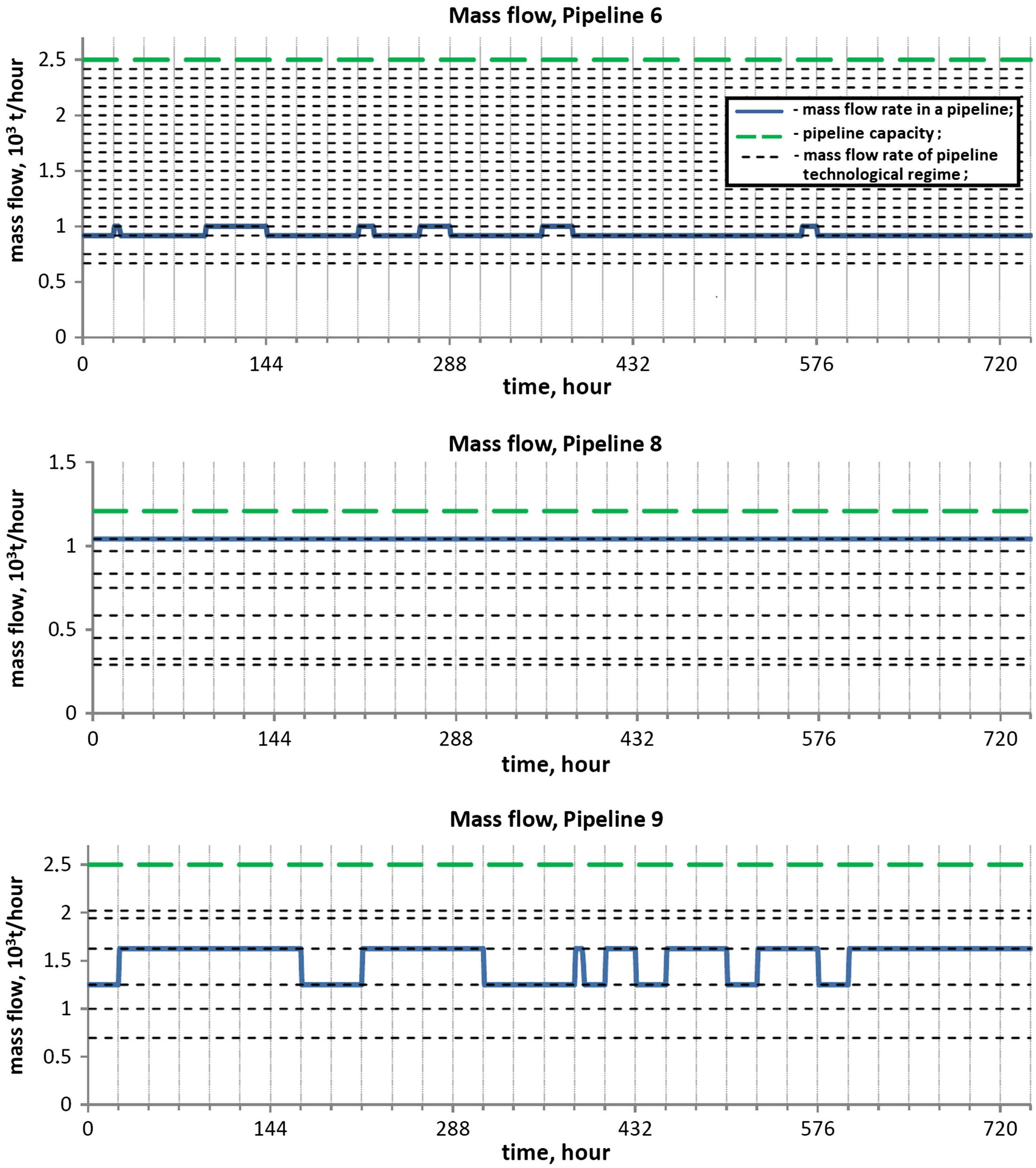


**Fig. 3.** Oil mass-flow rates in pipelines located between tank terminals.

$\theta$th time step; $a_s = A_s - \sum_{j=1}^{J_s} \sum_{\theta}^{T} q_{sj}^{\theta} t_{sj}^{\theta}$ is the amount of oil left with suppliers; $b_c = B_c - \sum_{j=1}^{J_c} \sum_{\theta}^{T} q_{cj}^{\theta} t_{jc}^{\theta}$ is the amount of oil that was not delivered to consumers; and $N_4$ and $N_5$ = weighting factors.

Equality constraints $b(x)$ and inequality constraints $c(x)$ for each and every pipeline from the example are shown in the Appendix.

Thus, using the proposed approach, the task of calculating the schedule of oil cargo flows in a branched pipeline system was solved for the described input data. The results of the solution are most visibly displayed as graphs showing the change in the mass-flow rate of oil pumping through a pipeline over time (Figs. 2–4), as well as the change in the amount of oil in tank terminals (Fig. 5) and oil properties at control points (Fig. 6).

Fig. 2 shows mass-flow rates over time in pipelines coming from suppliers (Pipelines 1, 2, and 4). The mass-flow rate in Pipeline 3 is unambiguously tied with the mass-flow rate in Pipeline 2. The mass-flow rate in Pipeline 2 is shown before the withdrawal at Pipeline 3.

Flow rates that are not from the set of fixed technological regimes and stopped state in Pipelines 1, 2, and 4 are due to the schedule of maintenance work (Table 4).

Fig. 3 shows the graphs of mass-flow rates from time in intermediate pipelines, which connect tank terminals to each other: Pipelines 6, 8, and 9. For pipes without fixed technological regimes (Fig. 1), the mass-flow rate is limited only by their capacity.

Mass-flow rates diagrams for Pipeline 5, 7, and 10, through which oil is delivered to consumers, are shown in Fig. 4. Based on the calculation results, all suppliers pumped the amount of oil specified in the transportation schedule (Table 1), and all consumers received the amounts of oil indicated in Table 2. The difference between the total mass of oil supplied to the pipeline system and amount of oil delivered to consumers is 10,000 t. According to the solution, this difference remained in the tank terminals.

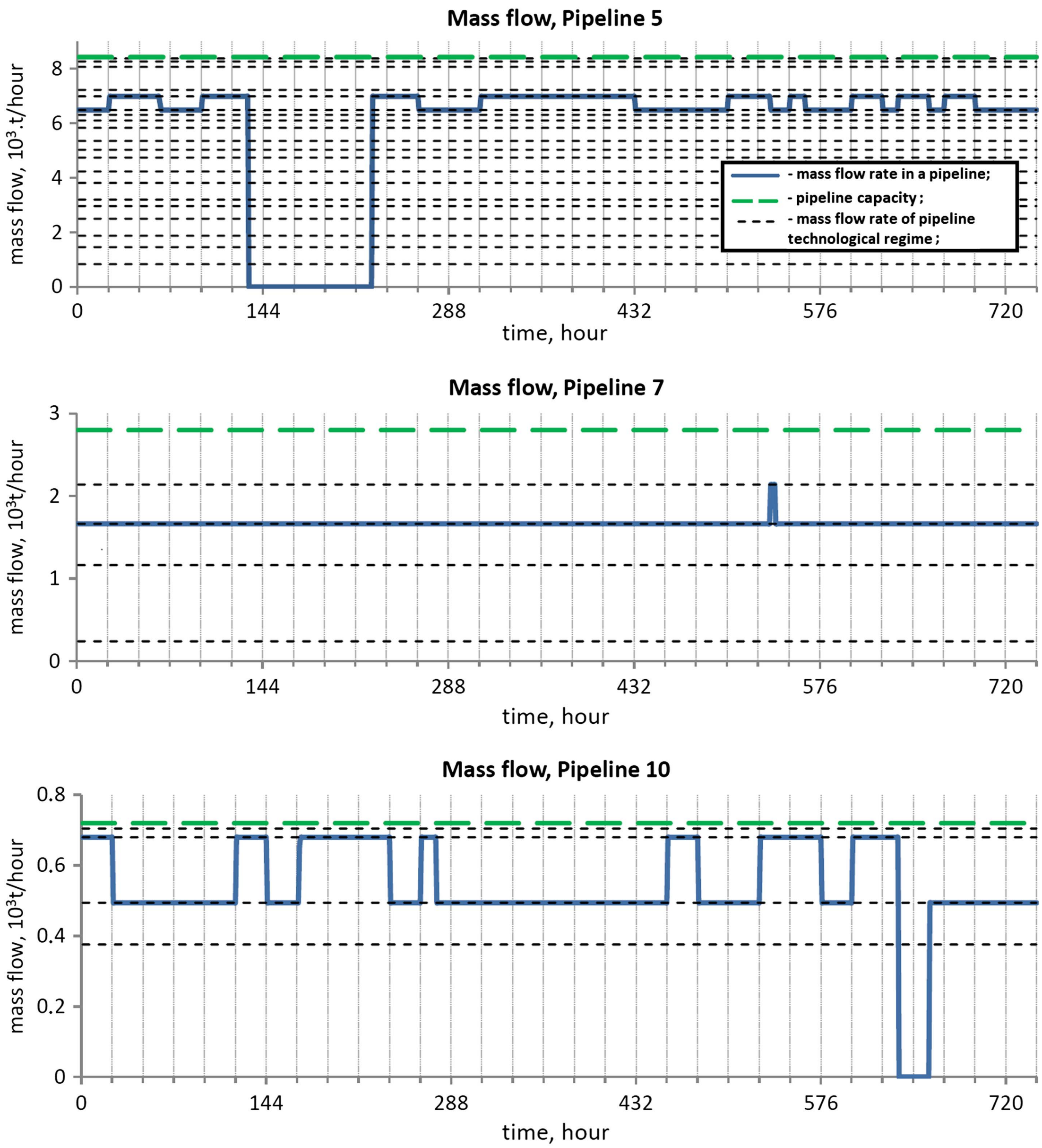


**Fig. 4.** Oil mass-flow rates in pipelines going to consumers.

In addition to graphs of mass-flow rates in pipelines, the solution result is shown in form of graphs of the amount of oil in all tank terminals from time (Fig. 5). During the whole period of the calculation, none of the tanks was re-emptied or overflowed, i.e., $M_{\min}^{T_i} \le M_{T_i}^{\theta} \le M_{\max}^{T_i}$ for each time step.

Oil properties at Consumers 3–6 will actually be determined by the value of properties in Tank terminal 3. Properties at Consumer 1 will be equal to the properties at Supplier 1. Therefore, two graphs are presented in Fig. 6: the mass fraction of sulfur from time for Consumer 2 and Tank terminal 3. Graphs of oil density will look similar. The density for all consumers did not exceed 880 $kg/m^3$ during the entire pumping period.

**Table 7.** Schedule of maintenance works for the pipeline system

| Pipeline | Start hour (h) | Duration, $T_{\text{plan}}$ (h) | Pipeline capacity, $q_{i\text{Plan}}^{\theta}$ (thousands of t/day) |
|---|---|---|---|
| 1 | 156 | 96 | 0 |
| 2 and 3 | 489 | 72 | 60 |
| 2 and 3 | 564 | 24 | 0 |
| 4 | 492 | 72 | 0 |
| 5 | 156 | 96 | 0 |
| 10 | 660 | 24 | 0 |

Using the graphs in Figs. 2–6, it is possible to (1) make a daily oil transportation schedule for a month, (2) determine the transition map for the technological regimes of the pipelines in the whole pipeline system, (3) determine the mixing of oil at the outlet from the tank terminals to the pipelines and in the tank terminals themselves, and (4) control the technological pumping process.

Computational time for a test case with a 1-h time step and 1-month (744 h) time period was 10 min using a computer with following characteristics: Windows 10 operating system, 32 GB RAM, and Intel core i7 (two cores) processor. The Octave programming language was used.

## Conclusion

The article described a new approach to oil-flow scheduling for a certain period with a given sampling step in a branched pipeline

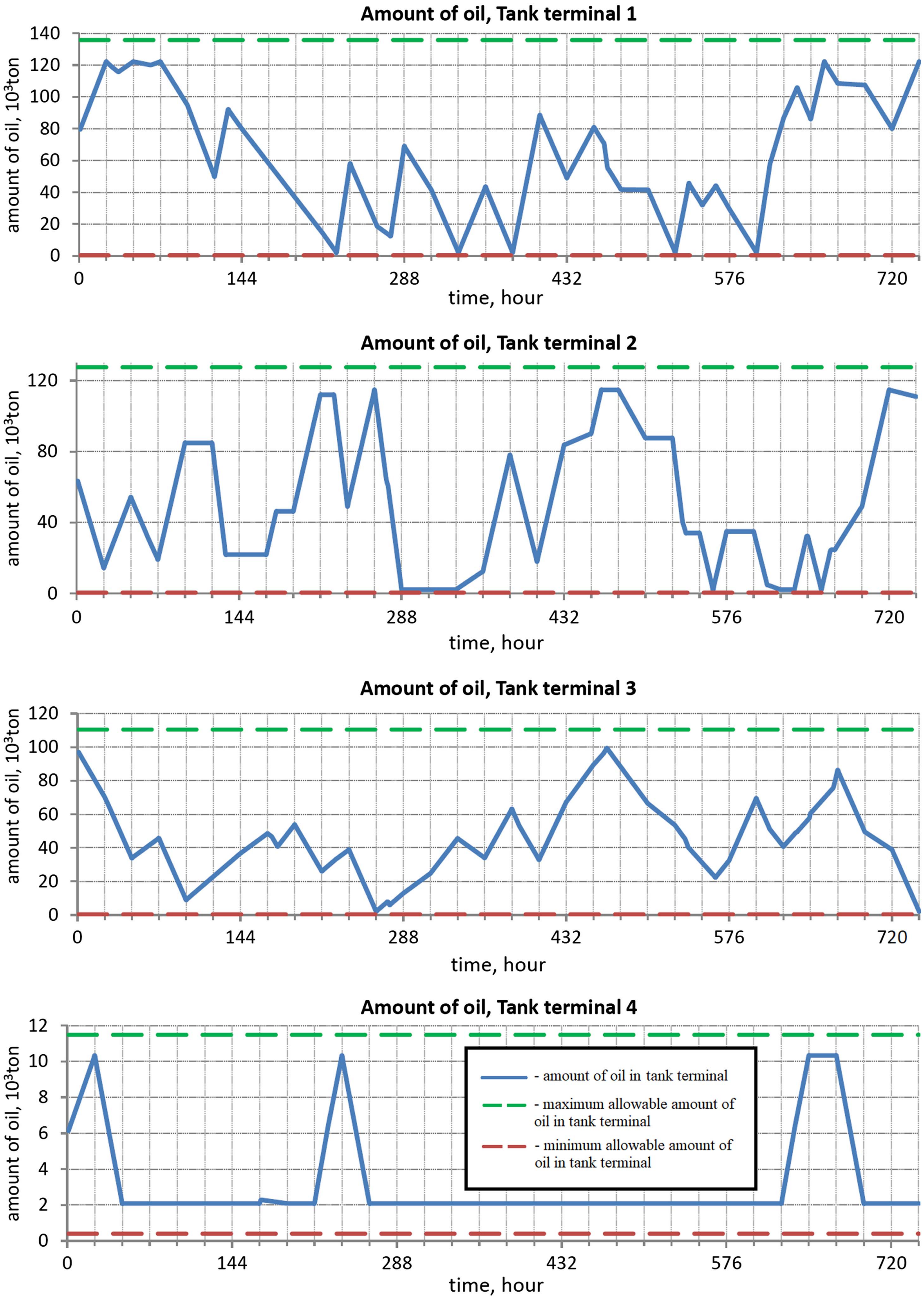


**Fig. 5.** Amount of oil in the tank terminals.

system, taking into account the fulfillment of the requirements for the quality of oil received by consumers, maintenance work, pipeline technological regimes, and the amount of oil in tank terminals.

The main advantages of this approach are

- use of a strict mathematical algorithm that can solve the problem without initial solution;
- ability for the method to be adapted for branched oil pipeline systems with tank terminals and discrete technological regimes in pipelines;
- ability of the approach to take into account oil transportation and properties distribution in pipelines as well as constraints on the oil properties at consumers; and



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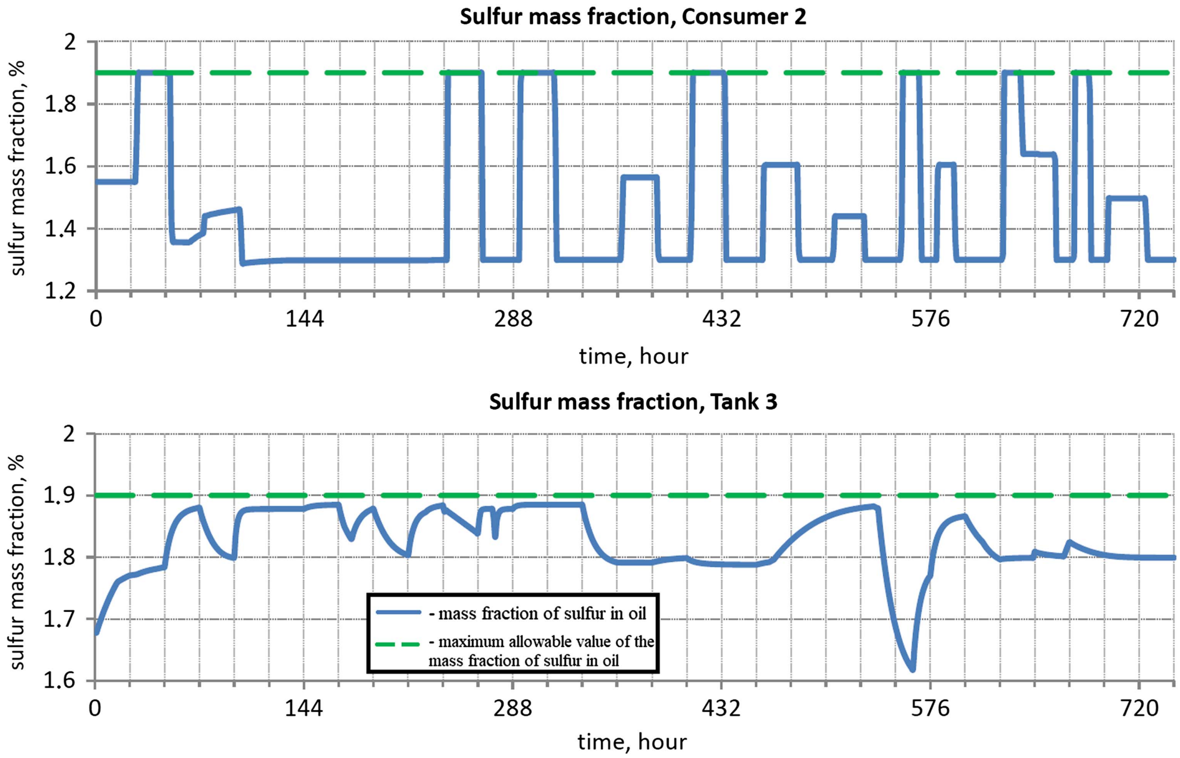


**Fig. 6.** Mass fraction of sulfur in oil.

- computation time that allows use of this approach in pipeline operations control centers to calculate transportation of oil and oil products in branched pipeline systems.

With this approach, oil cargo flows were calculated in a branched pipeline system taking into account all the conditions and limitations set by the user for the maximum value of oil volume in tank terminals, pipeline capacity, and properties of oil received by consumers, among others. Calculation results for the described pipeline system per month with a 1-h time sampling have been given. Values were obtained for oil pumping through the pipeline system (oil volume in tank terminals, technological conditions in pipelines, and oil properties at controlled points, among others) at each time step. The approach is quite universal and can be modified to take into account various conditions for scheduling oil flows in a particular branched pipeline system.

The article has described a new approach that was used as a test program to optimize cargo oil flows in branched pipeline systems in the Transneft Company.

## Appendix. Constraints for the Given Example of a Branched Pipeline System

### Mass Balance

$$\sum_{j=1}^{J_1} q_{1j}^{\theta} t_{1j}^{\theta} = q_1^{\theta} T_{\text{step}} + q_2^{\theta} T_{\text{step}} \quad \text{for all } \theta \text{ from 1 to 744}$$

$$\sum_{j=1}^{J_5} q_{5j}^{\theta} t_{5j}^{\theta} = q_3^{\theta} T_{\text{step}} + q_4^{\theta} T_{\text{step}} \quad \text{for all } \theta \text{ from 1 to 744}$$

$$\sum_{j=1}^{J_8} q_{8j}^{\theta} t_{8j}^{\theta} = q_6^{\theta} T_{\text{step}} + q_9^{\theta} T_{\text{step}} \quad \text{for all } \theta \text{ from 1 to 744}$$

$$\sum_{j=1}^{J_9} q_{9j}^{\theta} t_{9j}^{\theta} = q_7^{\theta} T_{\text{step}} + q_8^{\theta} T_{\text{step}} \quad \text{for all } \theta \text{ from 1 to 744}$$

### Time Balance

$$\sum_{j=1}^{J_i} t_{ij}^{\theta} = T_{\text{step}} \quad \text{for all } \theta \text{ from 1 to 744 and for all } I \text{ from 1 to 10}$$

### Constraints on the Oil Mass in Tank Terminals

$$M_{\min}^{T_1} < M_{T_1}^{\theta-1} - \sum_{j=1}^{J_6} q_{6j}^{\theta} t_{6j}^{\theta} + q_2^{\theta} T_{\text{step}} - q_3^{\theta} T_{\text{step}} < M_{\max}^{T_1}$$

for all $\theta$ from 1 to 744

$$M_{\min}^{T_2} < M_{T_2}^{\theta-1} + \sum_{j=1}^{J_3} q_{3j}^{\theta} t_{3j}^{\theta} - q_4^{\theta} T_{\text{step}} < M_{\max}^{T_2} \quad \text{for all } \theta \text{ from 1 to } T$$

$$M_{\min}^{T_3} < M_{T_3}^{\theta-1} + \sum_{j=1}^{J_6} q_{6j}^{\theta} t_{6j}^{\theta} + \left( \sum_{j=1}^{J_2} q_{2j}^{\theta} t_{2j}^{\theta} - \sum_{j=1}^{J_3} q_{3j}^{\theta} t_{3j}^{\theta} \right) + \sum_{j=1}^{J_4} q_{4j}^{\theta} t_{4j}^{\theta}$$
$$- q_5^{\theta} T_{\text{step}} - \sum_{j=1}^{J_7} q_{7j}^{\theta} t_{7j}^{\theta} - \sum_{j=1}^{J_8} q_{8j}^{\theta} t_{8j}^{\theta} - \sum_{j=1}^{J_9} q_{9j}^{\theta} t_{9j}^{\theta} < M_{\max}^{T_3}$$

for all $\theta$ from 1 to 744

$$M_{\min}^{T_4} < M_{T_4}^{\theta-1} + q_6^{\theta} T_{\text{step}} + q_7^{\theta} T_{\text{step}} - \sum_{j=1}^{J_{10}} q_{10j}^{\theta} t_{10j}^{\theta} < M_{\max}^{T_4}$$

for all $\theta$ from 1 to 744

### *Constraints on the Oil Properties*

$$s_{\min}^{c_2} \leq \frac{\frac{(M_{T_1}^{\theta-1} s_{T1}^{\theta-1} + q_2^\theta T_{\text{step}} s_{s1}^\theta)}{M_{T_1}^{\theta-1} + q_2^\theta T_{\text{step}}} q_3^\theta T_{\text{step}} + \frac{(M_{T_2}^{\theta-1} s_{T2}^{\theta-1} + \sum_{j=1}^{J_3} q_{3j}^\theta t_{3j}^\theta s_{s2}^\theta)}{M_{T_2}^{\theta-1} + \sum_{j=1}^{J_3} q_{3j}^\theta t_{3j}^\theta} q_4^\theta T_{\text{step}}}{q_3^q T_{\text{step}} + q_4^\theta T_{\text{step}}} \leq s_{\max}^{c_2} \quad \text{for all } \theta \text{ from 1 to 744}$$

$$s_{\min}^{T_3} \leq \frac{M_{T_3}^{\theta-1} s_{T_3}^{\theta-1} + \sum_{j=1}^{J_6} q_{6j}^\theta t_{6j}^\theta s_{s1}^\theta + \sum_{j=1}^{J_4} q_{4j}^\theta t_{4j}^\theta s_{s3}^\theta + (\sum_{j=1}^{J_2} q_{2j}^\theta t_{2j}^\theta - \sum_{j=1}^{J_3} q_{3j}^\theta t_{3j}^\theta) s_{s2}^\theta}{M_{T_3}^{\theta-1} + \sum_{j=1}^{J_6} q_{6j}^\theta t_{6j}^\theta + \sum_{j=1}^{J_4} q_{4j}^\theta t_{4j}^\theta + (\sum_{j=1}^{J_2} q_{2j}^\theta t_{2j}^\theta - \sum_{j=1}^{J_3} q_{3j}^\theta t_{3j}^\theta)} \leq s_{\max}^{T_3} \quad \text{for all } \theta \text{ from 1 to 744}$$

Above mentioned constraints is used just for the objective function without $\alpha$ and $\beta$.

### *Other Constraints*

$$0 \leq q_k^\theta \leq Q_k \quad \text{for all } \theta \text{ from 1 to 744 and all } k \text{ from 1 to 9}$$

$$0 \leq t_{ij}^\theta \leq T_{\text{step}} \quad \text{for all } \theta \text{ from 1 to 744, for all } I \text{ from 1 to 10, and for all } j \text{ from 1 to } J_i$$

## Notation

*The following symbols are used in this paper:*

$C$ = number of consumers;
$c$ = index of consumers in a pipeline system;
$f(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, T_{\text{step}})$ = function of energy costs for oil pumping through the pipeline system;
$I$ = number of pipelines with fixed set of technological regimes in system;
$i$ = index of a pipeline with fixed set of technological regimes;
$J_i$ = number of regimes in the $i$th pipeline;
$j$ = index of the technological regime;
$K$ = number of pipelines without fixed set of technological regimes in a system;
$k$ = index of a pipeline without fixed set of regimes;
$M_{T_i}^\theta$ = oil mass in $T_i$th tank terminal at the $\theta$th time step;
$N$ = number of points which are junctures of several pipelines;
$N_1, N_2, N_3, N_4, N_5$ = weighting factors in the objective function $\varphi(x)$;
$n$ = index of the control point (point where flows mix);
$\rho_n^\theta$, $s_n^\theta$ = density/mass fraction of sulfur (%) at the control point $n$ at the $\theta$th time step;
$\rho_i^\theta$, $s_i^\theta$ = density/mass fraction of sulfur (%) of flowing oil in the $i$th pipeline;
$Q_k$ = capacity of the $k$th pipeline;
$q_{ij}^\theta$ = mass-flow rate in the $i$th pipeline for the $j$th technological regime at the $\theta$th time step;
$q_k^\theta$ = mass-flow rate in the $k$th pipeline;
$S$ = number of suppliers;
$s$ = index of suppliers in a pipeline system;
$T$ = period of time when the transportation of pipeline system is optimized;
$T_i$ = index of a tank terminal;
$T_{\text{step}}$ = duration of a time step;
$t_{ij}^\theta$ = pumping time in the $i$th pipeline for the $j$th technological regime at the $\theta$th time step;
$\alpha, \beta$ = oil properties functions in objective function $\varphi(x)$;
$\gamma(q_{ij}^\theta, t_{ij}^\theta, q_k^\theta, T_{\text{step}})$ = discrepancy function of the total amount of oil actually delivered by suppliers and received by consumers from the amount specified by schedule;
$\theta$ = index of the time step;
$\varphi(x)$ = objective function; and
$\vartheta(t_{ij}^\theta, T_{\text{step}})$ = function that describes the number of transitions between regimes in a pipeline system.

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